Lec 15

Monday, February 13, 2023 1:26 PM

Recap

- selection, fitting, and validation
- selection of a model = hypothesis class of function
- fitting – finding the best parameter set for a selected model
- validation – test selected and fitted model on a unseen dataset

Modeling: Gaussian processes

$$y = f(x) + \varepsilon$$

$$f(x) = w^T x \quad \text{(linear model)}$$

$$= w^T \phi(x) \quad \text{(polynomial } \phi(x) = [1\ x\ x^2 \cdots])$$

$$\varepsilon \sim N(0, \sigma^2)$$

$$p(y|x,w) = \prod_{i=1}^{n} p(y_i|x_i,w) \Leftarrow$$

$$= \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(y_i - w^T x)^2}{2\sigma^2}\right)$$

Gaussian : sum, multiply, margnilization (over variable) conditional distribution.

$\Downarrow$

Gaussian

$$= N(w^T X, \sigma^2 I)$$

$\boxed{w \sim N(0, \Sigma_p)}$ → a sample would be a $p \times p$ matrix

$$\begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix}$$

$w_{11} = N(\mu, \sigma^2)$

$W \sim N(0, \Sigma_p)$

$$p(w|y,X)$$

$$= p(y|X,w) \times p(w)$$

$$= N(\bar{w}, \Sigma_w)$$

How are we going to predict at a unknown value x_* given our model $f(x_*)$ given (X, y)

$$p(f(x_*)|X,y,x_*)$$

$$= \int \underbrace{p(f_*|x_*,w)}_{\text{model}}\ \underbrace{p(w|X,y)}_{\text{posterior}}\, dw$$

$$= N(\bar{w}_*, \Sigma_{w*})$$

Gaussian processes :

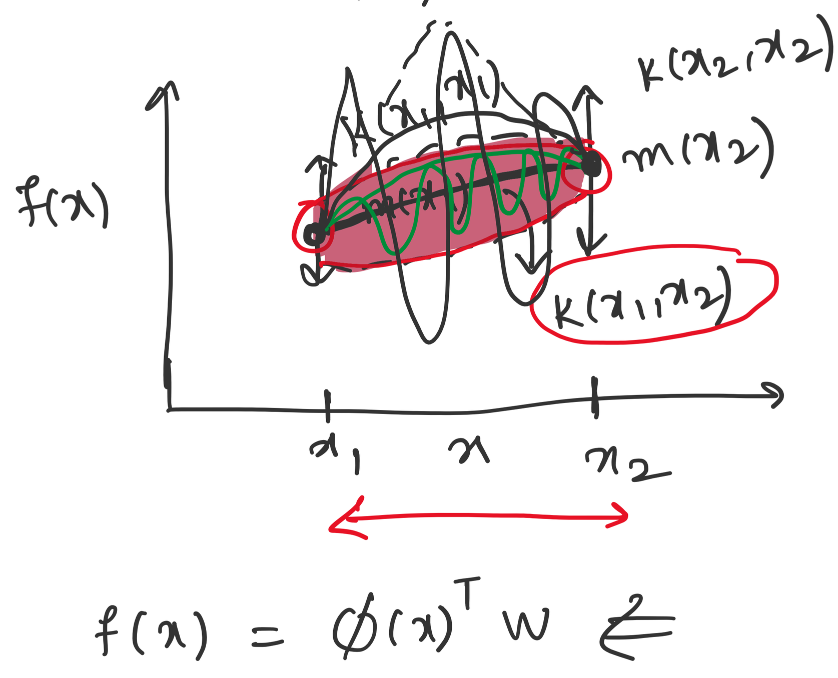
$$f = w^T x$$

$$f = GP(\underbrace{m(x)}_{\text{mean function}}, \underbrace{k(x,x')}_{\text{covariance function}})$$

$$m(x) = ax + b$$

$$= a \quad (a=0)$$

$K(x,x')$ = smooth



$$f(x) = \phi(x)^T w \Leftarrow$$

$$\mathbb{E}[f(x)] = \mathbb{E}[\phi(x)^T w]$$

$$= \phi(x)^T\, \mathbb{E}[w] = m(x)$$

$$\Rightarrow \mathbb{E}[f(x), f(x')] = \phi(x)^T\, \mathbb{E}[w w^T]\, \phi(x)$$

$$= k(x, x')$$

$$\mathbb{E}[(f(x) - f(x'))(f(x) - f(x'))^T]$$

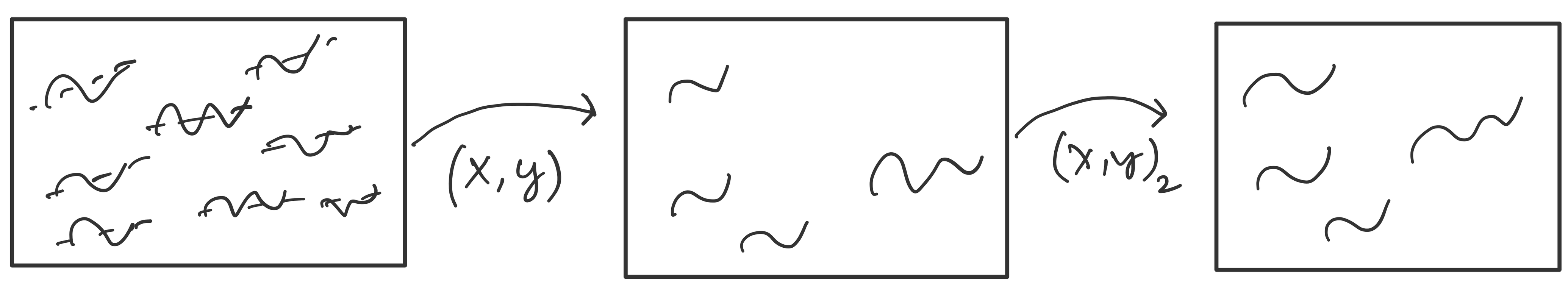
$m(x) = 0$

$k(x,x') \Leftarrow$ GP choices is the covariance. (prior knowledge)

- smooth, smoothness defined by length scale
- , sharp / peaky functions.

$$f(x_1) \longleftrightarrow f(x_2)$$

$$f(x) \sim GP(m(x), k(x,x'))$$



$p(f)$ being the model given data