Lec06

Wednesday, January 18, 2023 1:27 PM

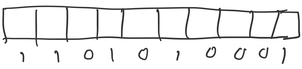
- No class on Fri 20
- Assignment 1 is due Mon, 23, Jan

Recap

- prior based estimation MAP (maximum a-posteriori)

$$\mathcal{L}(\mu,\sigma) = \underbrace{p(\mu)}_{\text{prior}}\ \underbrace{p(y|\mu)}_{\text{conditional}}$$

- Hypothesis testing



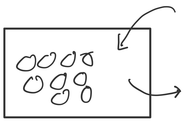
$y_1\ y_2 \cdots \cdots y_{20}$

$$\text{gain} := \bar{y}(i=1) - \bar{y}(i=0)$$

- known distribution

Today: Random sample method
Random design method

- $[y_1, y_2, \cdots, y_{20}]$



$y_i \rightarrow$ 20 such draws

$[y_1, y_2, \cdots, y_{20}]$

(Null hypothesis) $\Leftarrow \bar{y}(i=1) - \bar{y}(i=0)$

Alternative $\rightarrow$ They are from different boxes

$$\text{Var}(\bar{y}(i=1)) = \frac{\sigma^2}{n_1} \qquad \text{var}(\bar{y}(i=0)) = \frac{\sigma^2}{n_0}$$

$$\text{Var}\left(\bar{y}(i=1) - \bar{y}(i=0)\right) = \frac{\sigma^2}{n_0} + \frac{\sigma^2}{n_1}$$

$+(-\bar{y})$

$$\text{deviate } z = \frac{\bar{y}(i=1) - \bar{y}(i=0) - \delta}{\sigma\sqrt{\frac{1}{n_0} + \frac{1}{n_1}}}$$

$\delta = 0 \rightarrow$ Null hypothesis

How can we estimate σ (access to samples)

Substitute sample variance s for σ

$$s = \frac{(\text{diff to mean})^2}{\text{degrees of freedom}}$$

$\bar{y}(i=1)$, $\bar{y}(i=0)$, n_0, n_1

$$s = \frac{\sum (y(i=0) - \bar{y}(i=0))^2 + \sum (y(i=1) - \bar{y}(i=1))^2}{n_0 + n_1 - 2}$$

"t" deviate $t = \dfrac{\text{numer}}{s(\cdot)}$ $\quad (s \leftarrow \sigma)$

$\uparrow$

Student's t-distribution

Prob(t) > threshold $\rightarrow$ $\delta = 0 \rightarrow$ Null hypothesis cannot be discredited

Random design method

$y_1, y_2, y_3, \cdots, y_{20}$

✓current	1	0	0	⋯	0	$\rightarrow$ $\bar{y}(i=1) - \bar{y}(i=0)$
random label	0	0	1	⋯	1	gain_0

$\vdots$

$N = {}^{20}C_{10} = \dfrac{20!}{10!\,10!}$

y_1
$\vdots$
y_N