Lec 03

Monday, January 9, 2023 1:19 PM

Assignment 1 (due 01/23) is available on canvas and course website

Recap of lec 02

Population vs Sample

location	μ	$\bar{y}$
spread	σ	s

probability density $= f \rightarrow$ probability $= f \times h$

- random sample : each sample has equal chance of getting picked

Single variable (vs) Multivariable/variates

Normal distribution $\rightarrow$ Central limit effect

"experimental error tends to be symmetric"

$$P(y) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(y-\mu)^2}{2\sigma^2}\right)$$

probability density

$$\int_{-\infty}^{\infty} p(y)\, dy = 1$$

z (standardized variate of y) $= \frac{y-\mu}{\sigma}$

$\mathbb{E}[z] = 0$, $\sigma(z) = 1$

z – discounts for magnitude effects

score for each variate based $p(z)$

↳ z-score, normalization score

$$P(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) \rightarrow \mathcal{N}(\mu, \sigma)$$

symbol

$z \sim \mathcal{N}(0,1)$

variate

↳ Normal distribution

variate (mutiple numbers) $\boxed{y = [y_1, y_2, \cdots, y_k]}$

k-dimensional vector (components)

Covariance : i, j define Σ_{ij}

Matrix Σ of size $k \times k$

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ 1 & & & \\ 2 & & & \\ 3 & & & \end{array} \quad \begin{bmatrix} \sigma_1 & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_2 & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_3 \end{bmatrix} \qquad k = 3$$

location of y_k : $\mathbb{E}[y] = [\mathbb{E}[y_1], \cdots, \mathbb{E}[y_k]]_{1\times k} = \mu$

$$\Sigma_{ij} = \mathbb{E}[(y_i - \mu)(y_j - \mu)]$$

eigen values of Σ_{ij} are all positive; then Σ is called a positive definite matrix

A matrix A is mutipled by a vector x:

$Ax \rightarrow$ scaling, rotating vector x

eigen vectors, eigen values

$\Downarrow$

can only scale your vectors

$$Ax = \lambda x$$

multivariate normal distribution

$$P(y) = \frac{1}{\sqrt{(2\pi)^k |\Sigma|}} \exp\left(-\frac{1}{2}(y-\mu)^T \Sigma^{-1} (y-\mu)\right)$$

For a 2D case : y_1, y_2

$\rightarrow \mu = [0.5, -0.5] = (\mathbb{E}[y_1], \mathbb{E}[y_2])$

$$\Sigma = \begin{bmatrix} 2.0 & 0.01 \\ 0.01 & 1.0 \end{bmatrix}_{2\times 2}$$

$y_1 \sim \mathcal{N}(0.5, 2.0)$